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General Relativity as a (constrained) Yang-Mills's Theory and a Novel Gravity with Torsion.

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We show that General Relativity (GR) with cosmological constant *may be formulated* as a rather simple constrained $SO(D-1,2)$ (or $SO(D,1)$)-Yang-Mills (YM) theory. Furthermore, the spin connections of the Cartan-Einstein formulation for GR appear as solutions of a genuine $SO(D-1,1)$-YM.

We also present a theory of gravity with torsion as the most natural extension of this result. The theory comes out to be strictly an YM-theory upon relaxation of a suitable constraint. This work sets out to enforce the close connection between YM theories and GR by means of a new construction.

I. INTRODUCTION

There exists a great deal of attempts to formulate GR as a YM-type theory or, in general, as a gauge theory [1–7,10,8,9,11,12]. However, there is not yet a simple and conclusive result that establishes this connection very neatly.

Hehl et al [11] consider the Poincare group as the local symmetry group, and the basic dynamical variables of GR are obtained from the gauge fields (the connection) on a principal bundle over spacetime.

Mac Dowell and Mansouri [6] proposed a gauge theory of gravity based on the group $SO(3,2)$ (or $SO(4,1)$), for the first time. The Poincare group is obtained by the Wigner-Inönü contraction [13].

Other authors, Stelle-West [7] and Gotzes-Hirshfield [9] worked also along the same stream, in particular Stelle-West [7], recover GR by imposing a constraint in the action explicitly.

There has been an increasing revival and interest in this type of formulation in order to find 11d-SUGRA from algebras corresponding to 12-dimensional theories (F-theories) [14–17].

These approaches, however, have some disadvantages:

. They are gauge formulations, but they do not have a *genuine* YM-structure.

This means that the equations are of the Maxwell type, i.e they derive from an action proportional to the square of the gauge field-strength two-form. This is an important fact in order to implement a universal quantization scheme, similar to the one adopted for the other forces of Nature.

. They are restricted to D=4.

. They do not show how a genuine *YM-current* must be consistently related to the energy-momentum tensor.

In our work, we focus our attention on the equations of motion (of a YM's theory) rather than insisting on an analysis based upon the action. We start off with equations manifestly different from the ones of the approaches referred to above.

This approach succeeds in solving the points mentioned above. Furthermore, it shows clearly *where is* the real difference between GR and an YM-theory; this relies on a single constraint which has an extremely simple interpretation: the torsion-free condition.

Remarkably, this constraint reduces a second order theory (YM) to a first order one, the so-called Einstein-Cartan formulation of GR. The YM structure assumes an internal symmetry group which is broken by that constraint, leading to $SO(1,3)$ as the internal residual gauge group.

This paper is organized according to the following outline:

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In Section 2, we briefly introduce the Einstein-Cartan formalism and the main definitions of an YM-structure are established; next, in Section 3, it is shown that spin connections are solutions of $SO(D)$-YM, with an interesting form for the sources. Calculations in this direction have been done, in a different context, mainly for the purpose of numerical calculations [18–20].

The main result, the YM-formulation of GR, is presented in Section 4, where an appealing formulation of a theory of gravity with torsion arises naturally.

Finally, our Concluding Remarks are collected in Section 5.

The results presented below hold for an arbitrary D, but we particularize for $D = 4$ and a Lorentzian signature to have in mind Einstein-Cartan GR-theory, though it is not necessary.

II. EINSTEIN-CARTAN FORMALISM FOR GR AND THE SPIN CONNECTION AS AN YM VARIABLE.

In this work, we shall use the abstract index notation [21]; namely, a tensor of type (n, m) shall be denoted by $T^{a_1.....a_n}_{b_1.....b_m}$, where the latin index stand for the numbers and types of variables the tensor acts on and not as the components emelves on a certain basis. Then, this is an object having a basis-independent meaning. In contrast, greek letters label the components, for example $T^{\mu\nu}_{\alpha}$ denotes a basis component of the tensor T^{ab}_{c}. We start off with the Cartan's formalism of GR. We introduce [21] an orthonormal basis of smooth vector fields $(e_\mu)^a$, satisfying

$$(e_\mu)^a (e_\nu)_a = \eta_{\mu\nu}, \tag{1}$$

where $\eta_{\mu\nu} = diag(-1, 1, 1, 1......, 1)$. In general, $(e_\mu)^a$ is referred to as *vielbein*. The metric tensor is expressed as

$$g_{ab} = (e^\mu)_a (e^\nu)_b \eta_{\mu\nu}. \tag{2}$$

From now, component indices $\mu, \nu, ..$ will be raised and lowered using the flat metric $\eta_{\mu\nu}$ and the the abstract ones, $a, b, c...$ with space-time metric g_{ab}.

We define now the *Ricci rotation coefficients*, or *spin-connection*,

$$(w_{\mu\nu})_a = (e_\mu)^b \nabla_a (e_\nu)_b, \tag{3}$$

where $w_{a\mu\nu}$ is antisymmetric which, together with (1, is equivalent to the compatibility condition

$$\nabla_a g_{bc} = 0 \tag{4}$$

. From (3), we have

$$\nabla_a \; e^\mu{}_b = -w^{\mu\nu}{}_a e_{\nu\, b}. \tag{5}$$

Taking the antisymmetric part,

$$\nabla_{[a} \; e^\mu{}_{b]} = -w^{\mu\nu}{}_{[a} e^\alpha{}_{b]} \eta_{\nu\alpha}. \tag{6}$$

We have adopted the convention of anti- symmetrization:$(...)_{[ab]} = ((...)_{ab} - (...)_{ba})/2$.

In the original Einstein's formulation of GR, the
connection is assumed to be torsion-free, this is expressed by:

$$\partial_{[a} \; e^\mu{}_{b]} = -w^{\mu\nu}{}_{[a} e^\alpha{}_{b]} \eta_{\nu\alpha}. \tag{7}$$

The components of the Riemman's tensor in this orthonormal basis are given as follows

$$R_{ab}{}^{\mu\nu} := 2\partial_{[a} w^{\mu\nu}{}_{b]} + 2w^{\mu\rho}{}_{[a} w^{\sigma\nu}{}_{b]} \eta_{\rho\sigma}. \tag{8}$$

Equations (7) and (8) are the *structure equations* of GR in Cartan's framework.

The Einstein's equation is

$$e_\mu{}^a R_{ab}{}^{\mu\nu} = \kappa \; e_\nu{}^a T'_{ab} e, \tag{9}$$

where one has defined $T'_{ab} := T_{ab} + g_{ab}(T_{cd}g^{cd})/2$, T_{ab} being the energy momentum tensor.

Equations (5) and (9) sets up a system of coupled first-order non-linear equations for the variables (e, w) which determine [†] the dynamics of GR. Metric and covariant derivative result finally defined in terms of these variables as seen from (2) and (5).

This yields the so-called "Einstein-Cartan formalism"; we obtain thereby a first order Einstein-Hilbert's action which can be expressed as

$$S = \frac{1}{2\kappa^2}\int dx^D e R_{ab}{}^{\mu\nu} e_\mu{}^a e_\nu{}^b, \tag{10}$$

where $e = (-det\ g)^{1/2} = det\ (e^\mu{}_a)$. If we wish consider a non-vanishing cosmological constant, Λ, $R_{ab}{}^{\mu\nu}$ must be replaced by

$$R_{ab}{}^{\mu\nu} + \Lambda e^{[\mu}{}_a e^{\nu]}{}_b. \tag{11}$$

Finally, recall the YM's equations for a generic $SO(p,q)$ gauge field A_a^{AB}, where $A, B = 1, ...p+q$ label on the components of the gauge field and A_a^{AB} is a $(p+q)(p+q-1)/2$ collection of one forms ($A^{AB}_{\ a} = -A^{BA}_{\ a}$); They are the the dynamical variables of the theory whose equations of motion are second order.

We low and rise these internal index with the flat metric η_{AB}, which has p and q eigenvalues being 1 and -1 respectively.

Let us define the field-strength:

$$F_{ab}{}^{AB} := 2\partial_{[a}A^{AB}_{\ b]} + 2A^{AC}{}_{[a}A^{DB}{}_{b]}\eta_{CD}, \tag{12}$$

In a general curved Einstein's spacetime (with canonical connection ∇_a), the YM's equations are second order in the potentials:

$$\nabla^a F_{ab}{}^{AB} + 2A^{C[A\,a}F_{ab}{}^{B]}{}_C = J_b{}^{\ AB} \tag{13}$$

where $J_b{}^{\ AB}$ is the YM current. It is straightforward to show that this equation derives from a typical YM action, proportional to F^2.

This equation can be written shortly as

$$\mathcal{D}^a F_{ab}{}^{AB} = J_b{}^{\ AB}, \tag{14}$$

where we have defined the $SO(D)$ covariant derivative

$$\mathcal{D}_a K^{A_1....A_n} = \nabla_a K^{A_1...A_n} + \Sigma_{i=1}^n A^{A_i}{}_{C\,a}\ K^{A_1...A_{i-1}CA_{i+1}...A_n}, \tag{15}$$

$K^{A_1...A_n}$ being a spacetime tensor of arbitrary rank[‡].

Remark: Equation (12) together with (13) constitute the full structure of an $SO(p,q)$-YM theory up to a gauge-fixing, i.e; since the gauge invariances of equations (14), the fields A are not fully determined by these ones; thus, in order to solve a YM equation system, an additional (gauge) condition needs to be imposed.

We demonstrate below an important claim for this paper: the spin connection sector of the GR-solutions are solutions of a current YM theory too. This result will be critical for the construction of the next section.

Proposition **(2.1)**: The spin connection w_a of a D-dimensional smooth oriented Einstein-Cartan space time [§], constitutes an $SO(D-1,1)$-gauge field satisfying the $SO(D-1,1)$-Yang Mills equations (12),(13) on this (curved) space time. In other words, this means that if $w^{\mu\nu}{}_a$ is an antisymmetric field defined in terms of vielbein fields

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[†]Together with the antisymmetry condition for w_a.

[‡] Notice that two covariant derivatives appear: the background-covariant derivative, ∇_a and the YM's one, $\mathcal{D}_a$. Actually, $\mathcal{D}_a$ can be thought as the single one; recalling that ∇_a acts on spacetime (abstract) indices, the Christoffel symbols shall be taken into account.

[§]Of signature $(-,+,+,+)$.

and covariant (compatible) derivative by (3) such that the Einstein Equation (9) are satisfied, thus, the Yang-Mills equations (12),(13) on the corresponding space time, hold for the gauge field (A) taken to be ${w^{\mu\nu}}_a$.

Proof:

We have the field strength for the spin connection field defined as in (12):

$$ {R_{ab}}^{\mu\nu} := 2\partial_{[a} {w^{\mu\nu}}_{b]} + 2{w^{\mu\rho}}_{[a} {w^{\sigma\nu}}_{b]}\eta_{\rho\sigma}, \tag{16} $$

which is again an $SO(D-1,1)$ gauge invariant object, but now D agrees with the dimension of the spacetime (supposed to be a Lorentzian one). Henceforth, let us fix $D = 4$.

The Einstein-Cartan's equations will describe a subset of solutions of a YM theory, which has not invariance.

The Bianchi identity reads

$$ \nabla^a R_{bcda} + \nabla_{[b} R_{c]d} = 0, \tag{17} $$

but, using the symmetry properties of the Riemman's tensor $R_{bcda} = R_{dabc} = -R_{cbda}$, we find

$$ \nabla^a R_{adbc} - \nabla_{[b} R_{c]d} = 0 \tag{18} $$

Einstein's equation can be writen as $R_{ab} = \kappa T'_{ab}$, where $T'_{ab} := T_{ab} + g_{ab}(T_{cd} g^{cd})/2$ has been defined. Finally, we found an equation which holds for the on-shell GR [20]:

$$ \nabla^a R_{adbc} - \kappa \nabla_{[b} T'_{c]d} = 0. \tag{19} $$

On the other hand, the Riemann tensor is related to the YM-type field strength by

$$ {R_{ad}}^{\mu\nu} = R_{adcb} e^{\mu\,c} e^{\nu\,b}; \tag{20} $$

taking the divergence, it yields:

$$ \nabla^a {R_{ad}}^{\mu\nu} := [\nabla^a R_{adcb}] e^{\mu\,c} e^{\nu\,b} + R_{adcb}[\nabla^a e^{\mu\,c} e^{\nu\,b}] \tag{21} $$

Replacing (19) at the R.H.S. of this equation,

$$ \nabla^a {R_{ad}}^{\mu\nu} = \kappa[\nabla_{[c} T'_{b]d}] e^{\mu\,c} e^{\nu\,b} + R_{adcb}[\nabla^a e^{\mu\,c} e^{\nu\,b}]. \tag{22} $$

Let us concentrate on the last term; using the antisymmetry in c, b for the Riemann tensor, we may write:

$$ R_{adcb}[\nabla^a {e_\mu}^c {e_\nu}^b] = R_{ad\alpha\beta} {e^\alpha}_c {e^\beta}_b [\nabla^a {e_\mu}^c {e_\nu}^b]; \tag{23} $$

but, with the help of (3),

$$ {e^\alpha}_c {e^\beta}_b \nabla^a {e_\mu}^c {e_\nu}^b = {w^\alpha_\mu}^a\, \delta^\beta_\nu + {w^\beta_\nu}^a\, \delta^\alpha_\mu. \tag{24} $$

Replacing this in (23),

$$ R_{adcb}[\nabla^a e^c_\mu {e_\nu}^b] = {w^\alpha_\mu}^a\, R_{ad\alpha\nu} + {w^\alpha_\nu}^a\, R_{ad\alpha\mu}. \tag{25} $$

Finally, substituting it in (22), we have the remarkable result:

$$ \nabla^a R_{ab\mu\nu} - {w^\alpha_\mu}^a\, R_{ad\alpha\nu} - {w^\alpha_\nu}^a\, R_{ad\alpha\mu} = {j_b}^{\mu\nu}, \tag{26} $$

which has the form of the typical YM equation (13) with the "YM-current" defined as [20]:

$$ {j_a}^{\mu\nu} := \kappa[\nabla_{[c} T'_{b]a}] {e_\mu}^c {e_\nu}^b \tag{27} $$

This completes the demonstration.

III. EQUIVALENCE OF GR TO A (CONSTRAINED) $SO(3;2)$ (OR $SO(4;1)$)-YM THEORY AND INCLUSION OF TORSION.

The aim of this section is to show that GR (with cosmological constant) *can be written as* a YM theory plus certain constraints whose elimination led to a natural way to define the theory (GR) including torsion. This is the main construction of this paper.

We shall restrict ourselves to empty space to render more clear and evident some points; but the generalization to the case when matter is taken into account is straightforward and will be done at the end of the section.

Let us define this theory and prove that this is equivalent to GR.

Let M be a four-dimensional manifold with a smooth (oriented) metric (M, g_{ab}) of signature $(-,+,+,+)$. We shall also asume that, to each point $p \in M$, we can assign a real 5-dimensional vector space, V, equipped with a scalar product given by $\eta_{CD} := diag(-1,1,1,1,(-1)^s)$. This defines the group $SO(3+s;2-s)$ (where $s = \pm 1$), since this is the group that preserves the structure on V.

The dynamics is fully described by second order equations ** in the gauge variables A_a^{AB} (which for definition, are antisymmetric in A, B):

$$\nabla^a G_{ab}{}^{AB} + 2A^{C[A\,|a} G_{ab}{}^{B]}{}_C = J_b{}^{AB}, \tag{28}$$

where the field strength reads:

$$G_{ab}{}^{AB} := 2\partial_{[a} A^{AB}{}_{b]} + 2A^{AC}{}_{[a} A^{DB}{}_{b]} \eta_{CD}. \tag{29}$$

Note that for writing down these equations we have implicitly supposed the existence of a Riemannian metric g and a covariant derivative ∇. Without them, equations (28) would not make sense, however, below we will close the system by imposing relations such that this geometry structure will be given in terms precisely of the gauge variables.

Alternatively, it is more convenient to express this structure in the language described at the previous section, we are assuming the existence of (e, w), where e is defined trough

$$g_{ab} = (e^\mu)_a (e^\nu)_b \eta_{\mu\nu}, \tag{30}$$

and the spin-connection coefficients, $(w_{\mu\nu})_a$, are defined in the general case, i.e for *any* covariant derivative:

$$(w_{\mu\nu})_a = (e_\mu)^b \nabla_a (e_\nu)_b, \tag{31}$$

.

Then, we can define the torsion†† by

$$\theta_{ab}{}^\mu := \partial_{[a} e^\mu{}_{b]} + w^{\mu\nu}{}_{[a} e^\rho{}_{b]} \eta_{\nu\rho}. \tag{32}$$

Now, we make a *global* choice of the fifth basis element of V: U, defined satisfying, $\nabla_a U = 0$. Then, we define:

$$E^A{}_b := A^{A5}{}_b = A^A{}_{B\ b}\, U^B. \tag{33}$$

Let us use the greek letters to denote the first four components, ie $A = \mu, 5$, with $\mu = 1, ...4$.

The introduction of that vector is related to the Wigner-Inönü contraction which reduces $SO(3+s;2-s)$ to $ISO(3,1)$, the standard gauge group of GR [6,7,11]. Notice, however, that a contraction parameter has not yet been introduced and this will be not necessary in this construction.

Then, as previously announced, we write down the suplementar condition (constraint)

$$G_{ab}{}^{A\,5} = 0. \tag{34}$$

**The same ones that (13) and (12) with $d = 4$, $A; B = 1, ...5$.

††Notice that additional structure as the antisymmetry of the one forms, w_a, which implies the compatibility condition (4), together with the current torsion-free condition are not introduced *a priori* in this formulation, they shall be get from the equations defining the theory.

This is a first order equation relating the gauge fields; then, it constitutes a constraint for the above YM dynamical system. Notice that up this point, the theory, namely the YM-equations plus (34), manifestly appears to be $SO(1,3)$-gauge invariant.

Replacing (34) in the $A-5$ component of (28) -with $J_b{}^{AB}=0$-,

$$A^{C[A\,|a}G_{ab}{}^{B]D}\eta_{CD}=0; \tag{35}$$

taking $A=\mu$ and $B=5$, we find

$$[A^{C\mu\,a}G_{ab}{}^{5D}-A^{C5\,a}G_{ab}{}^{\mu D}]\eta_{CD}=0. \tag{36}$$

Since η_{CD} is diagonal, using again (34), we obtain

$$G_{ab}{}^{\mu\nu}A_{\mu 5}{}^{a}=0. \tag{37}$$

Finally, wqe shall relate the geometry variables with the YM-fields. Actually, this YM-type theory, coincides with GR once the identifications [6,7,11] are imposed:

$$\mu e^{\mu}{}_{a}=E^{\mu}{}_{a}, \tag{38}$$

$$w^{\mu\nu}{}_{a}=A^{\mu\nu}{}_{a}, \tag{39}$$

where μ is a parameter which has inverse length dimension related to the cosmological constant, as it shall become clearer later on. This parameter is introduced in order to give a dimensionless vielbein field, however, there is no some indication a priori of any scale of length in the theory.

From (39), w must be antisymmetric. This identification finally fixes the relation between the fields of theory (A_a^{AB}) and the background space time structure. Equation (34) is recognized as the first of the Cartan's structure equations, which expresses the non-torsion condition.

Replacing (38) into (34), A (or w) can be solved in terms of e_b^{ν}, in an Einstein-Cartan scheme. Then, in according with (32), we deduce that the torsion of the spacetime covariant derivative ∇, vanishes. This, plus the antisymmetry of w determines completely this connection, which results to be the canonical one.

Equation (37) with the identifications(39), (38), read as the (vacuum) Einstein's equation:

$$G_{ab}{}^{\mu\nu}e_{\mu}{}^{a}=0 \tag{40}$$

This is the (vacuum) Einstein's equation with a cosmological-constant term because

$$G_{ab}{}^{\mu\nu}:=2\partial_{[a}w^{\mu\nu}{}_{b]}+2w^{\mu\rho}{}_{[a}w^{\sigma\nu}{}_{b]}\eta_{\rho\sigma}+2(-1)^{s}\mu^{2}e^{[\mu}{}_{a}e^{\nu]}{}_{b} \tag{41}$$

ie

$$G_{ab}{}^{\mu\nu}:=R_{ab}{}^{\mu\nu}+2(-1)^{s}\mu^{2}e^{[\mu}{}_{a}e^{\nu]}{}_{b}, \tag{42}$$

this resembles (11) with $\Lambda=(-1)^{s}\mu^{2}$.

Notice furthermore that this (the Einstein's theory), is all the structure we can extract of the theory, in other words, the other YM equations do not introduce extra conditions. To show this, we shall use strongly the result of the previous section.

Going to the μ,ν-components of (28), we get

$$\mathcal{D}^{a}R_{ab}{}^{\mu\nu}+(-1)^{s}m^{2}\mathcal{D}^{a}(e^{[\mu}{}_{a}e^{\nu]}{}_{b})=0 \tag{43}$$

Notice that by virtue of (39) the full covariant divergence in (28) agrees with the one of the proposition **(2.1)** ($SO(1,3)$)

$$\mathcal{D}^{a}G_{ab}{}^{\mu\nu}=\nabla^{a}G_{ab}{}^{\mu\nu}+2A^{C[\mu\,|a}G_{ab}{}^{\nu]}{}_{C} \tag{44}$$

using (34),

$$\mathcal{D}^{a}G_{ab}{}^{\mu\nu}=\nabla^{a}G_{ab}{}^{\mu\nu}+2A^{\alpha[\mu\,|a}G_{ab}{}^{\nu]}{}_{\alpha}. \tag{45}$$

The term $\mathcal{D}^a(e^{[\mu}{}_a e^{\nu]}{}_b)$ vanishes using (31) -or equivalently, (5)-.

Thus, (42) reduces to the $SO(3,1)$-YM equation, which already has been proven -proposition **(2.1)**- to be identically satisfied by the fields e, w being solutions of the Einstein equation (empty) in the presence of a cosmological constant):

$$R_{ab}{}^{\mu\nu} e_\mu{}^a = \kappa T'_{(\Lambda)ab} e^{\nu\, a}, \tag{46}$$

which completes the proof of our claim.

The generalization to a non-trivial energy-momentum tensor, T_{ab}, is straightforwardly obtained by starting with an YM-theory with sources. In order to recover GR, the YM-current must be defined in terms of the general energy-momentum tensor:

$$J_{a5}{}^\mu := \kappa T_a^{'}\ {}^\mu, \tag{47}$$

where $T_a^{'}\ {}^\mu = T'_{ab} e^{\mu\, b}$.

Finally, we have consistency with the above results if the other components of the YM-current are defined to satisfy:

$$J_a{}^{\mu\nu} := \kappa[\nabla_{[c} T'_{b]a}] e_\mu{}^c e_\nu{}^b. \tag{48}$$

A. A gravity theory with torsion.

Torsion appears in a natural way in modern formulations of the gravitational theories [22]. This supports the framework discussed below (our final result).

Notice that, by relaxing the constraint (34), we are naturally led to a particularly elegant theory of gravity *with torsion*, which remarkably enough turns out to be an ordinary $SO(3+s; 2-s)$-YM. This theory is described as before by the dynamical equations:

$$\nabla^a G_{ab}{}^{AB} + 2A^{C[A\,|a} G_{ab}{}^{B]}{}_C = J_b{}^{AB}, \tag{49}$$

where

$$G_{ab}{}^{AB} := 2\partial_{[a} A_{b]}{}^{AB} + 2A^{AC}{}_{[a} A^{DB}{}_{b]}\ \eta_{CD}, \tag{50}$$

$$J_b{}^{AB} = (J_b{}^{\mu 5}; J_b{}^{\mu\nu}) = \kappa(T_a^{'}\ {}^\mu;\ [\nabla_{[c} T'_{b]a}] e_\mu{}^c e_\nu{}^b). \tag{51}$$

In order to describe gravitation, the identification constraints to be imposed are (38), (39); thus, the physical spacetime torsion is given by

$$\Theta := \mu^{-1} G_{ab}{}^{\mu 5}, \tag{52}$$

where we observe that the cosmological constant must be non-vanishing.

The modified Einstein's equation results from the $\mu - 5$ component of (28). It reads:

$$R_{ab}{}^{\mu\nu} e_\mu{}^a = -\mathcal{D}^a \Theta_{ab}{}^\mu + \kappa\ T_b^{'}\ {}^\mu, \tag{53}$$

The components $\mu - \nu$ of Equation constitute complementary ones, which are identities when the torsion is vanish.

IV. CONCLUDING REMARKS.

We conclude by stressing a remark: the meaning of the identification expressed by equations (38), (39). Formally, such an identification shall be looked upon as a constraint.

It has been argued in similar approaches that one can formulate GR without cosmological constant, by setting the appropriate limit $\mu \to 0$; in this case, the YM-group tends remarkably to the Poincare-Lorentz one via the well-known algebra contraction. Care is needed with this since point: in this limit, the structure underlying this approch appears to be singular as we can see in equations (38) and (52). These are two important issues and commonly, they are not remarked in the previous similar formulations.

It remains to be more deeply investigated the existence of exact solutions to YM theories starting from the particular ones well-known in GR. The issue of quantizing the theory in the presence of the constraint in the form presented here, is also a delicate and relevant matter to be pursued.

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